

Original Proposed Parameters

| Model Parameters | | | | | | | | | | Mean Reversion Speed | | Long Term Levels | |
|------------------|----------------|--------|--------|---------|---------|----------|---------------|----------|--------|----------------------|-------|------------------|--------|
| CIR Process | Theta | Kappa | Sigma | Lambda0 | Lambda1 | Gamma | Gamma + Kappa | LT State | Target | Calculated Target | | Actual | Target |
| CIR 1 | 0.2716 | 5.6773 | 0.0181 | -0.2515 | 5.3995 | 5.677311 | 11.35456 | 7.22% | 7.22% | 3.60 | 3.60 | 1-Year | 2.50% |
| CIR 2 | 0.0196 | 0.2520 | 0.0423 | -0.0025 | -0.0188 | 0.259034 | 0.511067 | 6.29% | 6.29% | 3.69 | 3.69 | 20-Year | 3.50% |
| CIR 3 | 0.0010 | 0.0000 | 0.0390 | 0.0011 | -0.0595 | 0.055149 | 0.055164 | 3.47% | 3.47% | 16.80 | 16.80 | | |
| | | | | | | | | | | | | Diff | 0.0% |
| Time | Shift Function | | | | | | | | | | | | |
| 0 | -0.1271 | | | | | | | | | | | | |

Revised Parameters

| Model Parameters | | | | | | | | | | Mean Reversion Speed | | Long Term Levels | |
|------------------|----------------|--------|--------|---------|---------|----------|---------------|----------|--------|----------------------|-------|------------------|--------|
| CIR Process | Theta | Kappa | Sigma | Lambda0 | Lambda1 | Gamma | Gamma + Kappa | LT State | Target | Calculated Target | | Actual | Target |
| CIR 1 | 0.2716 | 5.6773 | 0.0181 | -0.2586 | 5.3995 | 5.677311 | 11.35456 | 4.67% | 4.67% | 3.60 | 3.60 | Overnight | 2.25% |
| CIR 2 | 0.0196 | 0.2520 | 0.0423 | -0.0007 | -0.0188 | 0.259034 | 0.511067 | 6.96% | 6.96% | 3.69 | 3.69 | 1-Year | 2.50% |
| CIR 3 | 0.0010 | 0.0000 | 0.0390 | 0.0010 | -0.0595 | 0.055149 | 0.055164 | 3.33% | 3.33% | 16.80 | 16.80 | 20-Year | 3.50% |
| | | | | | | | | | | | | Diff | 0.0% |
| Time | Shift Function | | | | | | | | | | | | |
| 0 | -0.1271 | | | | | | | | | | | | |

What was the issue

The original proposal for converting long-term targets (e.g. MRPs) into GEMS parameters only adjusted formulas for two of the state variables. With the current Academy Model, those two targets drive the rest of the Yield curve's evolution. In the GEMS model, there is a third degree of freedom. Since we were not controlling this third item, it led to these unintended mean Yield curve shapes over time:

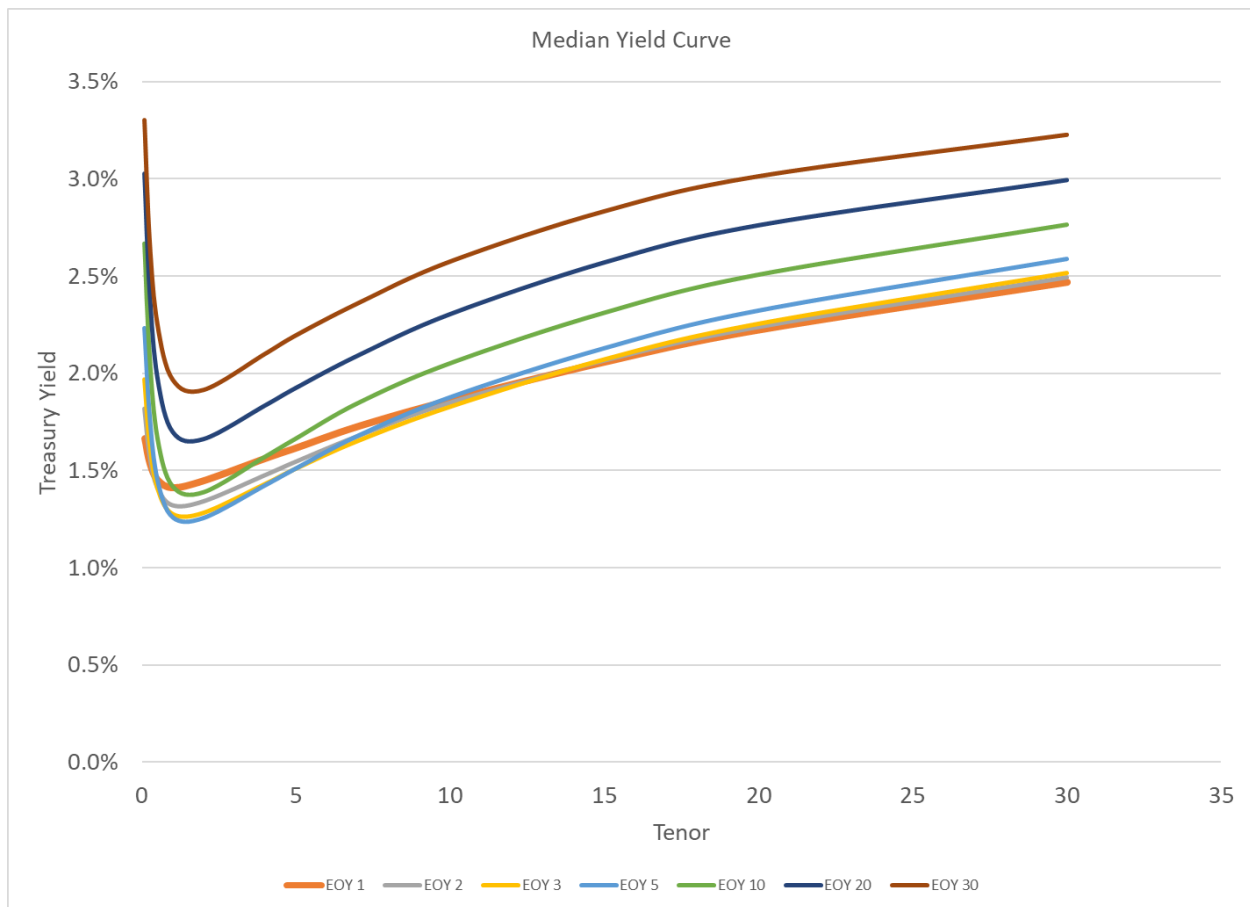


Figure 1: Projected Median Yield Curve at Future Periods with Original Proposed Parameters

The desire is to have the typical Yield curve is to have a standard shape (i.e. gradually increasing across all maturities). This can be accomplished by adding a third long-term target. In this revised set, we added a target for the Overnight Yield (i.e. 0-Month Tenor Yield in GEMS) that is 25 bps below. For this example, the 25 bps was based on what the current Academy model would produce for the Overnight Yield if the 1- and 20-Year Yields were at their long-term level. (Note: This is an estimate since the Academy model does not actually produce an Overnight Yield. In the final proposal, Conning would recommend setting this with A. actual historical data, similar to the way the other targets are set and B. could be switched to another short tenor, say the 1-Month Yield, if preferred.) This new set of targets leads to this set of comparable graphs:

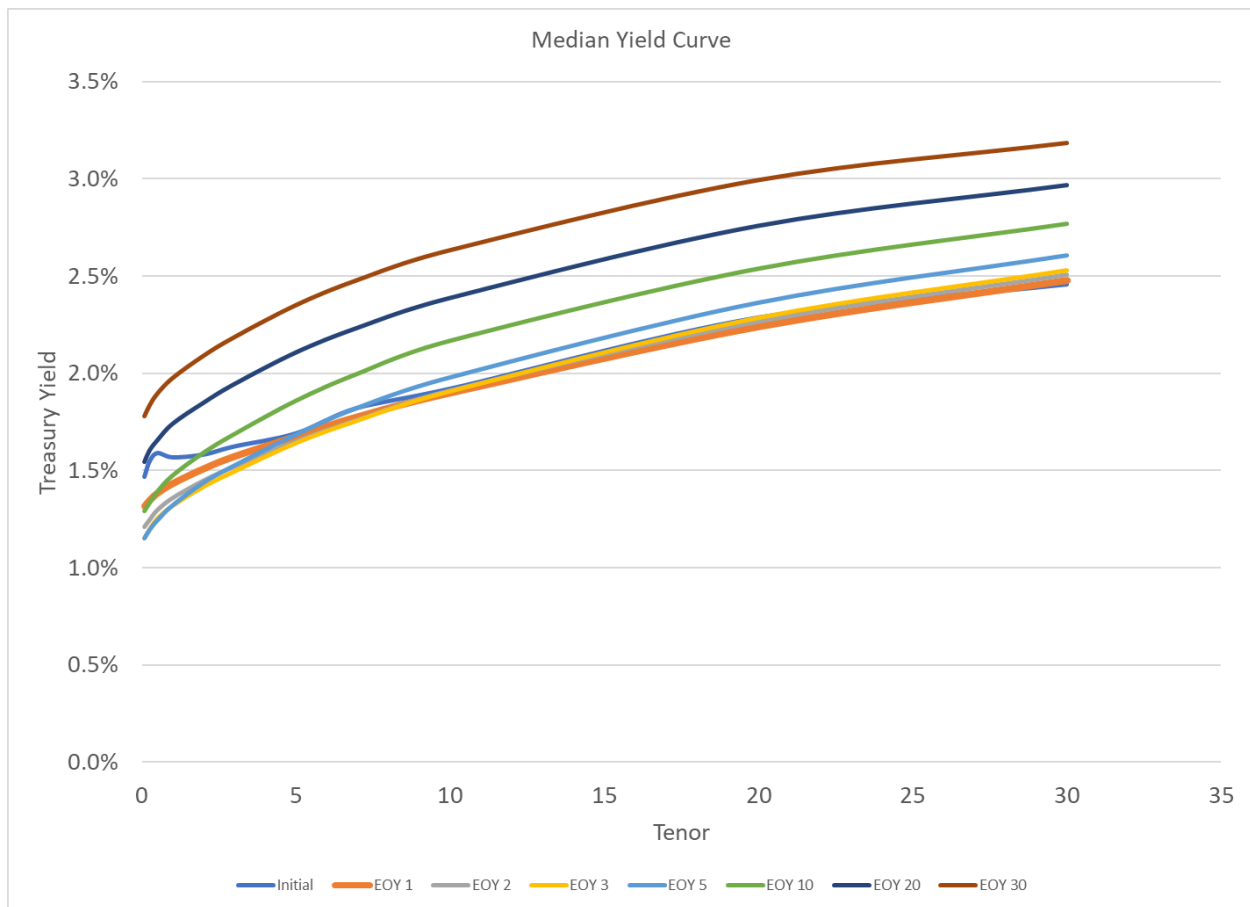


Figure 2: Projected Median Yield Curve at Future Periods with Revised Parameters

In particular, if we just focus on the EOY 30 values, we can see that the two curves cross at the original target points (i.e. the 1- and 20-Year Yields), but have very different extrapolations to the very short end

of the curve. Since the GEMS' Yield curves are smooth, this also has ramifications for the middle of the curve: the Revised Parameters produce higher yields in between the 1- and 20-year tenors.

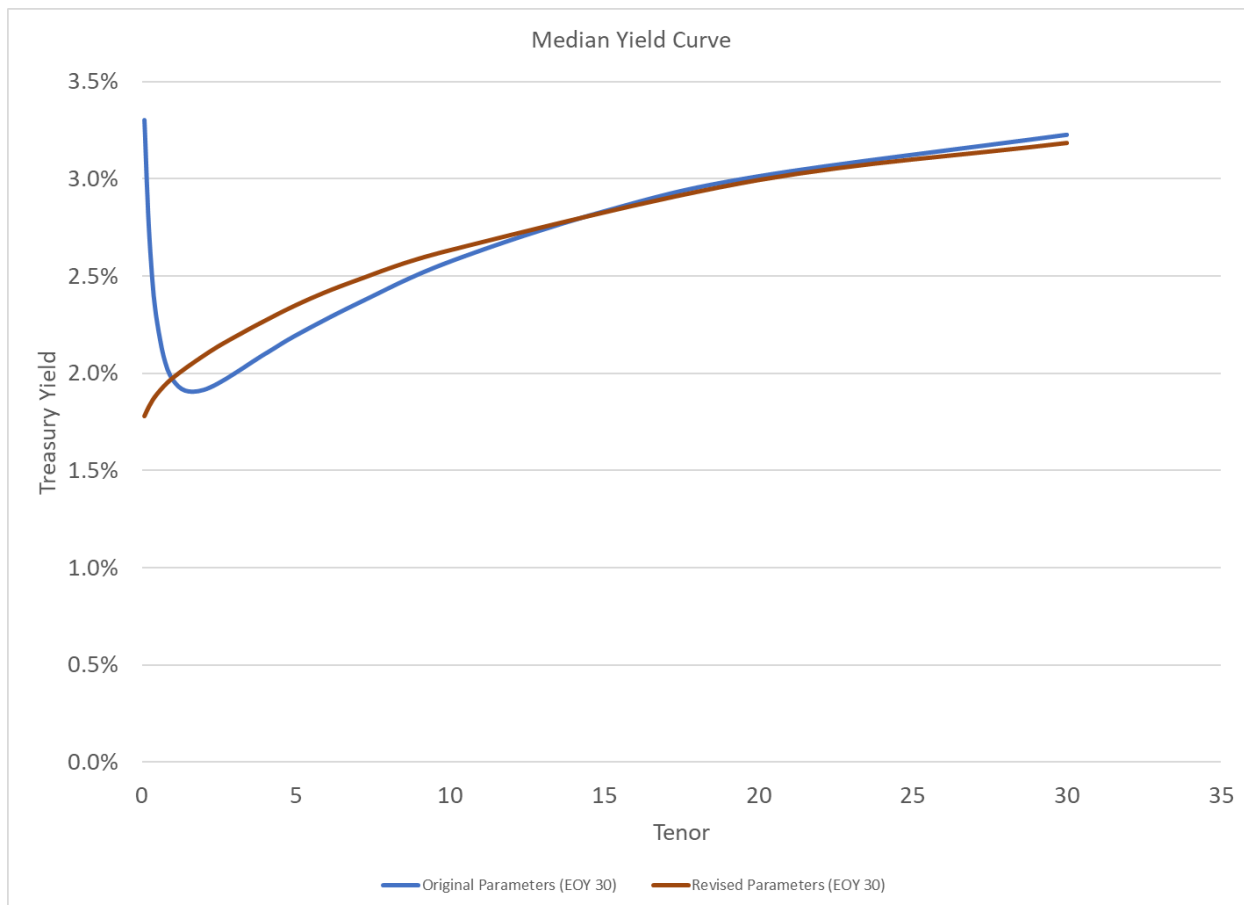


Figure 3: Comparison of EOY 30 Projected Median Yield Curve for two sets of Parameters

What changed? How did that impact the parameters?

| Model Parameters | | | | | | | | | | Mean Reversion Speed | | Long Term Levels | |
|------------------|----------------|--------|--------|---------|---------|----------|---------------|----------|--------|----------------------|--------|------------------|--------|
| CIR Process | Theta | Kappa | Sigma | Lambda0 | Lambda1 | Gamma | Gamma + Kappa | LT State | Target | Calculated | Target | Actual | Target |
| CIR 1 | 0.2716 | 5.6773 | 0.0181 | -0.2586 | 5.3995 | 5.677311 | 11.35456 | 4.67% | 4.67% | 3.60 | 3.60 | Overnight | 2.25% |
| CIR 2 | 0.0196 | 0.2520 | 0.0423 | -0.0007 | -0.0188 | 0.259034 | 0.511067 | 6.96% | 6.96% | 3.69 | 3.69 | 1-Year | 2.50% |
| CIR 3 | 0.0010 | 0.0000 | 0.0390 | 0.0010 | -0.0595 | 0.055149 | 0.055164 | 3.33% | 3.33% | 16.80 | 16.80 | 20-Year | 3.50% |
| Time | Shift Function | | | | | | | | | | | | Diff |
| 0 | -0.1271 | | | | | | | | | | | | 0.0% |

Conning added a third long-term target. Specifically, based on the current Academy's extrapolation procedure, Conning estimates that the Overnight Yield would be about 2.25% if the 1-Year was at 2.5% and the 20-Year was at 3.5%. These are not necessarily the final targets (e.g. the targets are actually expressed in different terms: the Overnight is a continuously compounded Spot Rate, while the other two are semi-annual Par Yields), rather it is meant as relatively easy example of how the process could be adjusted to eliminate the observed issues.

With that additional target, there is a change in the target long-term state variables which would achieve these values. Mathematically, we have 3 equations (i.e. the formulas for calculating the target points on the Yield Curve) in 3 unknowns (i.e. the long-term Target State variables). While the equations are complicated, especially for the 20-Year Yield, the fact that all three values are strictly increasing functions of the state variables ensures that there will be a unique answer to this process.

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| CIR 1 | 0.2716 | 5.6773 | 0.0181 | -0.2586 | 5.3995 | 5.677311 | 11.35456 | 4.67% | 4.67% | 3.60 | 3.60 | Overnight | 2.25% | 2.25% |
| CIR 2 | 0.0196 | 0.2520 | 0.0423 | -0.0007 | -0.0188 | 0.259034 | 0.511067 | 6.96% | 6.96% | 3.69 | 3.69 | 1-Year | 2.50% | 2.50% |
| CIR 3 | 0.0010 | 0.0000 | 0.0390 | 0.0010 | -0.0595 | 0.055149 | 0.055164 | 3.33% | 3.33% | 16.80 | 16.80 | 20-Year | 3.50% | 3.50% |
| Time | Shift Function | | | | | | | | | | | | Diff | 0.0% |
| 0 | -0.1271 | | | | | | | | | | | | | |

Once we have those targets, we adjust the **Lambda0** parameters so that their reversion level is equal to these target values. As mentioned in the **NAIC Technical Documentation - Interest Rates, DRAFT.pdf** that was distributed as material for the December 3, 2020 LATF call, the formula for the state variables has a drift term of: $(\text{Theta} + \text{Lambda0}) - (\text{Kappa} - \text{Lambda1}) * \text{Current State}$ (page 3). This formula will be at its steady state level when $\text{State} = (\text{Theta} + \text{Lambda0}) / (\text{Kappa} - \text{Lambda1})$. The formula in the **Lambda0** inverts this equation to find a revised value. (Note: $1 / (\text{Kappa} - \text{Lambda1})$ is also the Mean Reversion speed.)

| Model Parameters | | | | | | | | | Mean Reversion Speed | | | | Long Term Levels | | |
|------------------|----------------|--------|--------|---------|---------|----------|---------------|----------|----------------------|-------------------|-------|-----------|------------------|-------|--|
| CIR Process | Theta | Kappa | Sigma | Lambda0 | Lambda1 | Gamma | Gamma + Kappa | LT State | Target | Calculated Target | | Actual | Target | | |
| CIR 1 | 0.2716 | 5.6773 | 0.0181 | -0.2586 | 5.3995 | 5.677311 | 11.35456 | 4.67% | 4.67% | 3.60 | 3.60 | Overnight | 2.25% | 2.25% | |
| CIR 2 | 0.0196 | 0.2520 | 0.0423 | -0.0007 | -0.0188 | 0.259034 | 0.511067 | 6.96% | 6.96% | 3.69 | 3.69 | 1-Year | 2.50% | 2.50% | |
| CIR 3 | 0.0010 | 0.0000 | 0.0390 | 0.0010 | -0.0595 | 0.055149 | 0.055164 | 3.33% | 3.33% | 16.80 | 16.80 | 20-Year | 3.50% | 3.50% | |
| Time | Shift Function | | | | | | | | | | | | Diff | 0.0% | |
| 0 | -0.1271 | | | | | | | | | | | | | | |

One final note: once we have a decision on what the targets are, Conning will be writing up a full description of how those targets get converted into parameters for the GEMS Treasury model.